Optimal Scheduling of Capture Times in a Multiple Capture Imaging System

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Background

- High frame rate CMOS image sensors with non-destructive readout capabilities have been recently demonstrated
  - 352x288 10,000 f/s DPS (Kleinfelder ISSCC'01)
  - 272x260 480 f/s APS (Handoko VLSI Symposium'00)
- Applications to still and video imaging
  - Dynamic range enhancement via multiple capture (Yadid-Pecht IEEE-TED'97, Yang JSSC'99)
  - Motion blur restoration (Liu ICASSP'01)
  - Optical flow estimation (Lim ICIP'01)

Multiple Capture Example

τ

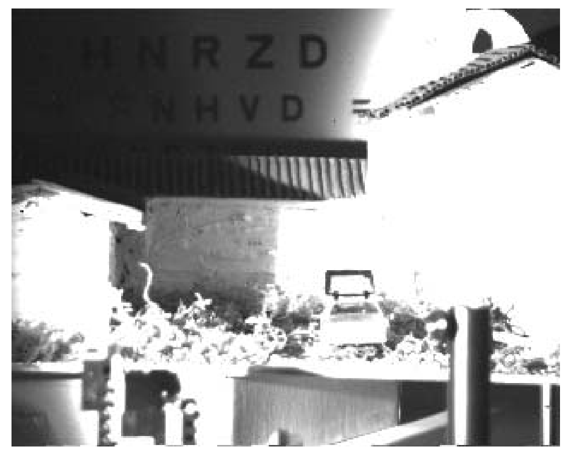
2τ

4τ

8τ

16τ

32τ



(Courtesy of J. DiCarlo)

High Dynamic Range Image

Last-Sample-Before-Saturation (LSBS) Algorithm: use each pixel's last sample before saturation with appropriate scaling

Optimal Capture Time Scheduling

- Uniformly and exponentially spaced capture times have been assumed based on certain implementation considerations (e.g., Yang SPIE'99, Liu SPIE'01)

- No systematic study of how capture times should be optimally scheduled

- Using optimal schedules less captures are needed to achieve the image quality requirements

  – Less computation, memory, power consumption

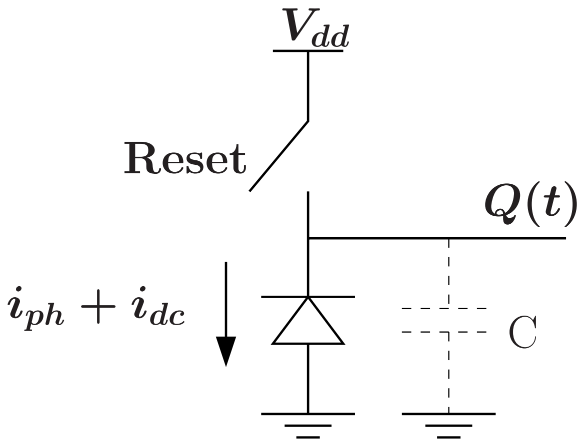
  – Less noise generated by multiple readouts

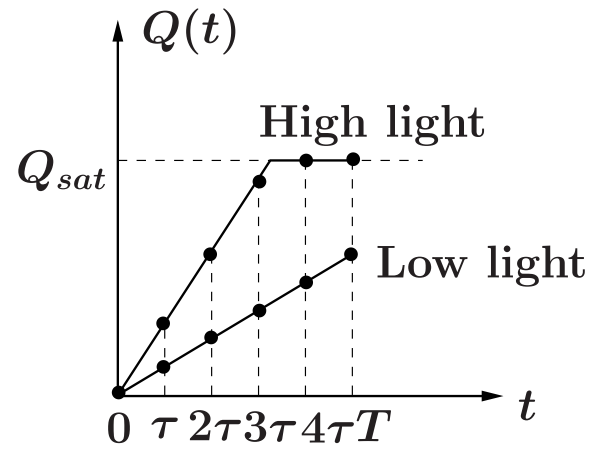
- To find optimal schedules, need information about scene illumination (same as auto-exposure)

This Work

- Assume complete a priori knowledge of scene illumination statistics, *i.e.*, probability density function (pdf)
- Formulate capture time scheduling as an average SNR maximization problem
- Find optimal schedules for uniform pdfs
- Discuss how to efficiently solve for piece-wise uniform pdfs

Sensor Noise Model and SNR

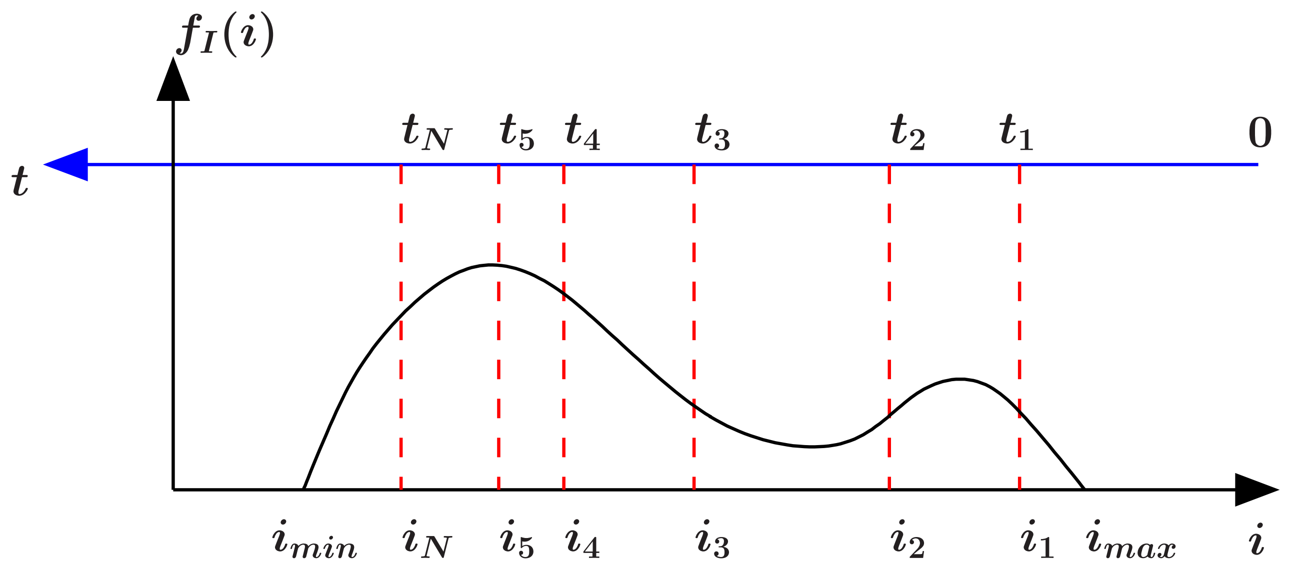




- $Q(T) = (i_{ph} + i_{dc})T + U(T) + V(T) + C$ for $Q(T) \leq Q_{sat}$
  - $U(T) \sim \mathcal{N}(0, q(i_{ph} + i_{dc})T)$: shot noise
  - $V(T) \sim \mathcal{N}(0, \sigma_V^2)$: readout noise
  - $C \sim \mathcal{N}(0, \sigma_C^2)$: reset and FPN noise
- $\text{SNR}(i_{ph}) = \dfrac{(i_{ph}T)^2}{q(i_{ph} + i_{dc})T + \sigma_V^2 + \sigma_C^2}$ for $i_{ph} \leq i_{max}$
- LSBS chooses the last non-saturating sample since it has the highest SNR

Capture Time Scheduling Problem

- Assume complete knowledge of the scene induced photocurrent pdf $f_I(i)$ over $(i_{\min}, i_{\max})$, find capture times $\{t_1, t_2, ..., t_N\}$ to maximize the average SNR



- Neglecting read noise and FPN, for every t_k, let $i_k = Q_{sat}/t_k$ be the corresponding maximum non-saturating photocurrent

- Finding $\{t_1, ..., t_N\} \equiv$ finding $\{i_1, ..., i_N\}$

Optimization Problem

- Using LSBS, SNR for photocurrent $i_{k+1} < i \leq i_k$ is now given by :

$$\mathrm{SNR}(i) = \frac{Q_{\mathrm{sat}} i}{q i_k}$$

- Given $f_I(i)$ and N, find $\{i_2, ..., i_N\}$ to maximize the average SNR

$$E\left(\mathrm{SNR}(i_2, ..., i_N)\right) = \sum_{k=1}^{N} \int_{i_{k+1}}^{i_k} \mathrm{SNR}(i) f_I(i)\, di,$$

subject to: $i_{\min} = i_{N+1} < i_N < \ldots < i_k < \ldots < i_1 = i_{\max}$

Optimal Scheduling for Uniform PDFs

- Given a uniform photocurrent pdf over the interval $(i_{\text{min}}, i_{\text{max}})$ and N, find $\{i_2, i_3, ..., i_N\}$ that maximizes the average SNR

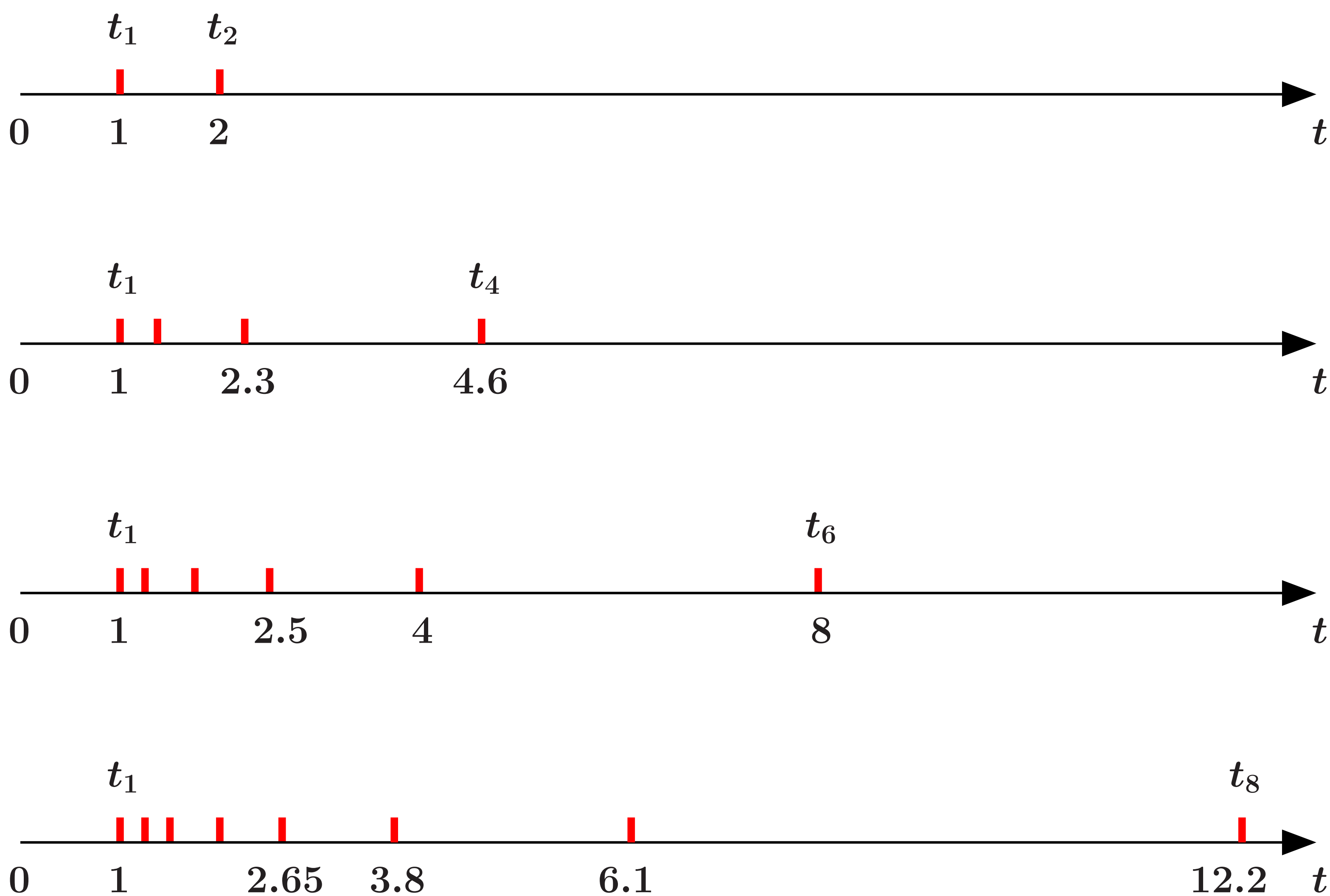
$$E\left(\text{SNR}(i_2, i_3, ..., i_N)\right) = \frac{Q_{\text{sat}}}{q(i_{\text{max}} - i_{\text{min}})} \sum_{k=1}^{N} (i_k - \frac{i_{k+1}^2}{i_k}),$$

subject to: $i_{\text{min}} = i_{N+1} < i_N < \ldots < i_k < \ldots < i_1 = i_{\text{max}}$

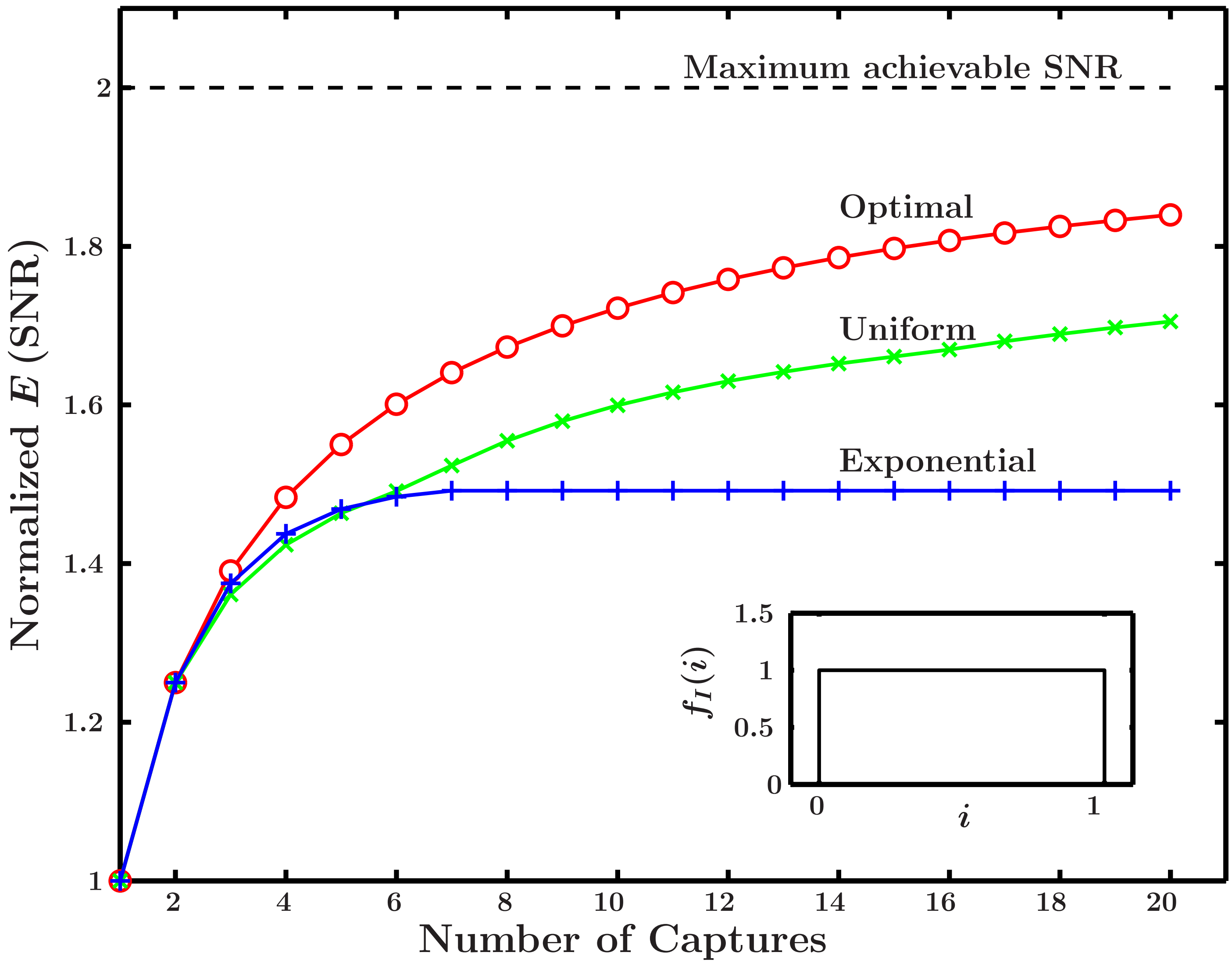
- $E(\text{SNR}(i_2, i_3, ..., i_N))$ is a concave function in $\{i_2, i_3, ..., i_N\}$

- Can efficiently find the global optimum using convex optimization techniques

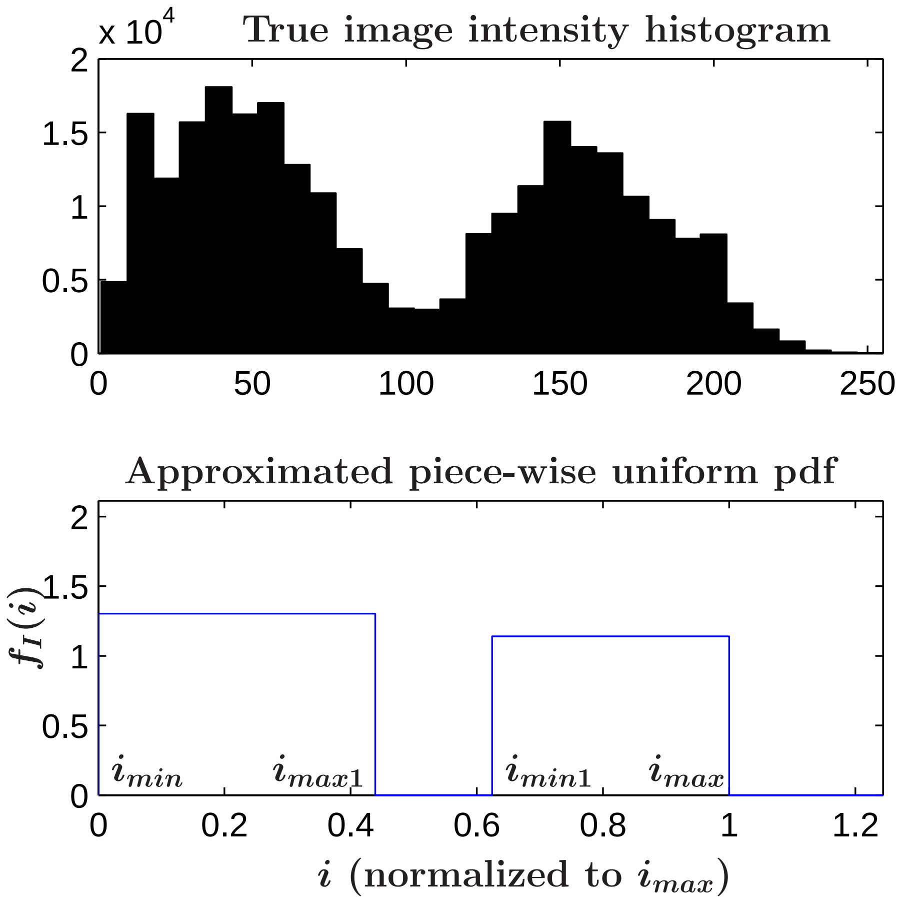
Optimal Capture Times for a Uniform PDF over $(0, 1)$



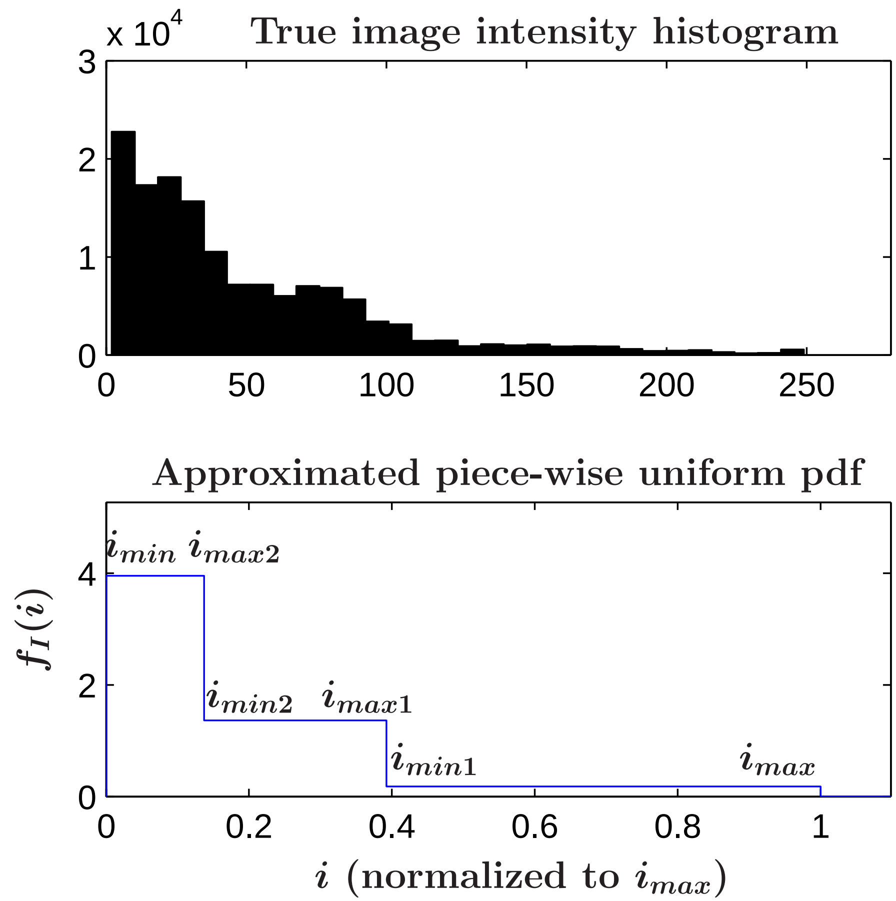
Performance Comparison of Optimal, Uniform, and Exponential (exponent = 2) Schedule



An Image with Approximated Two-Segment Piece-Wise Uniform PDF



An Image with Approximated Three-Segment Piece-Wise Uniform PDF



Scheduling for Two-Segment Piece-Wise Uniform PDFs

- First assume k out of total N captures are assigned to segment (i_{min1}, i_{max})

- Find $\{i_2, ..., i_N\}$ to maximize the average SNR

$$E\,(\mathrm{SNR}) = c_1 \sum_{j=1}^{k-1} (i_j - \frac{i_{j+1}^2}{i_j}) + c_1 (i_k - \frac{i_{\min 1}^2}{i_k}) + c_2 \frac{i_{\max 1}^2 - i_{k+1}^2}{i_k} + c_2 \sum_{j=k+1}^{N} (i_j - \frac{i_{j+1}^2}{i_j})$$

subject to: $0 \le i_{\min} = i_{N+1} < i_N < \ldots < i_{k+1} < i_{\max 1} \le i_{\min 1} \le i_k < \ldots < i_1 = i_{\max} < \infty$

- Solution is found by solving the above problem for $1 \le k \le N$

- $E\,(\mathrm{SNR}(i_2, ..., i_N))$ is concave for all $\{i_2, i_3, ..., i_N\}$ except i_k

- Non-concave term $c_2 i_{max1}^2 / i_k$ is convex

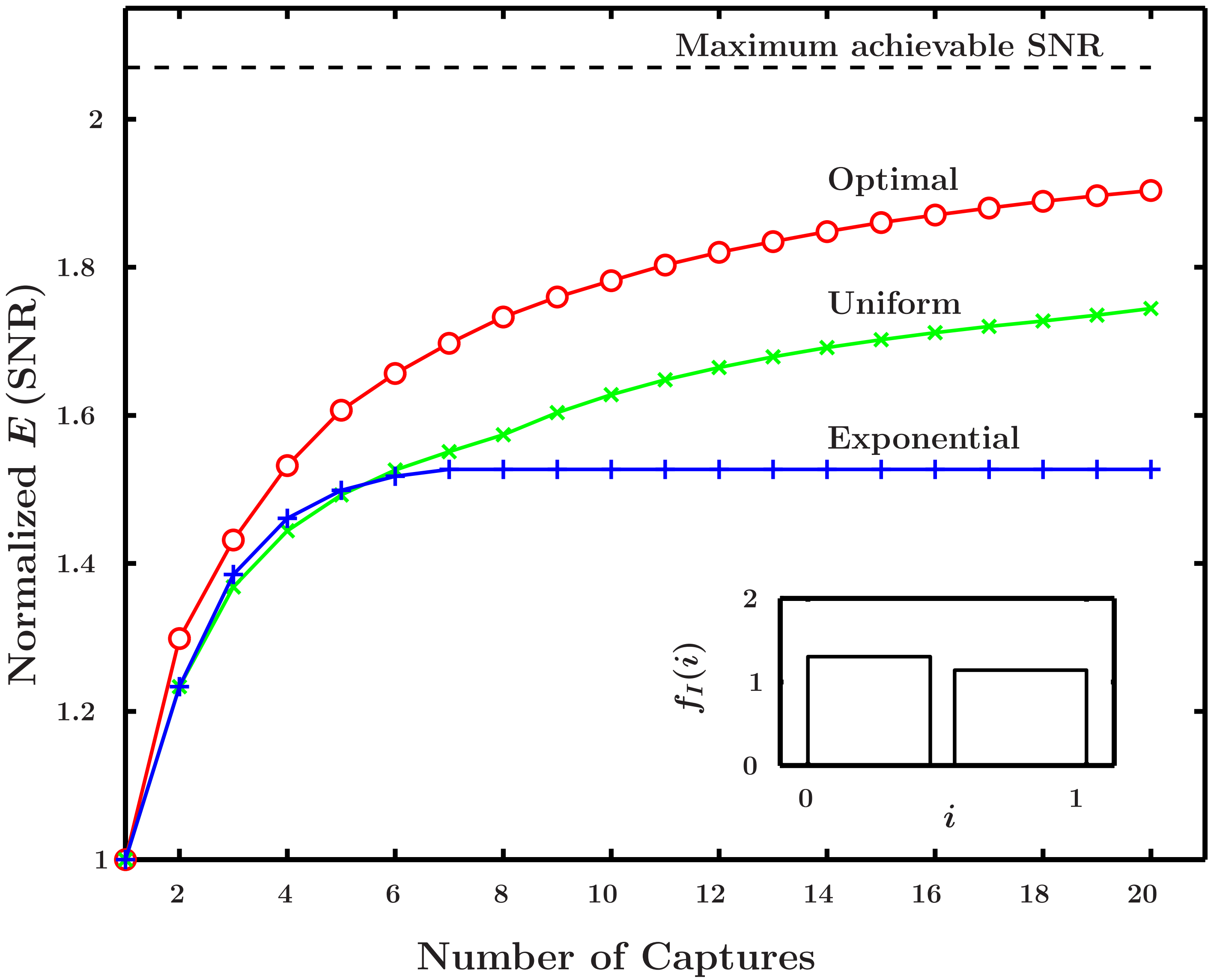
- $E(\mathrm{SNR}(i_2, ..., i_N))$ is a D.C. (Difference of Convex) function and solvable via D.C. optimization techniques

Scheduling for M-Segment Piece-Wise Uniform PDFs

- **For any M-segment piece-wise uniform pdf, the capture time schedule for N captures is found by**
  - **Assigning $\{k_1, k_2, \ldots, k_M\}$ captures to the M segments subject to $\sum_{j=1}^{M} k_j = N$**
  - **Finding the best schedule using D.C./global optimization techniques for the assignment**
  - **Choosing the best assignment and corresponding schedule that maximizes $E(\text{SNR})$**

- **We use Sequential Quadratic Programming (SQP) with multiple randomly generated initial conditions**

Performance Comparison of Optimal, Uniform, and Exponential (exponent = 2) Schedule



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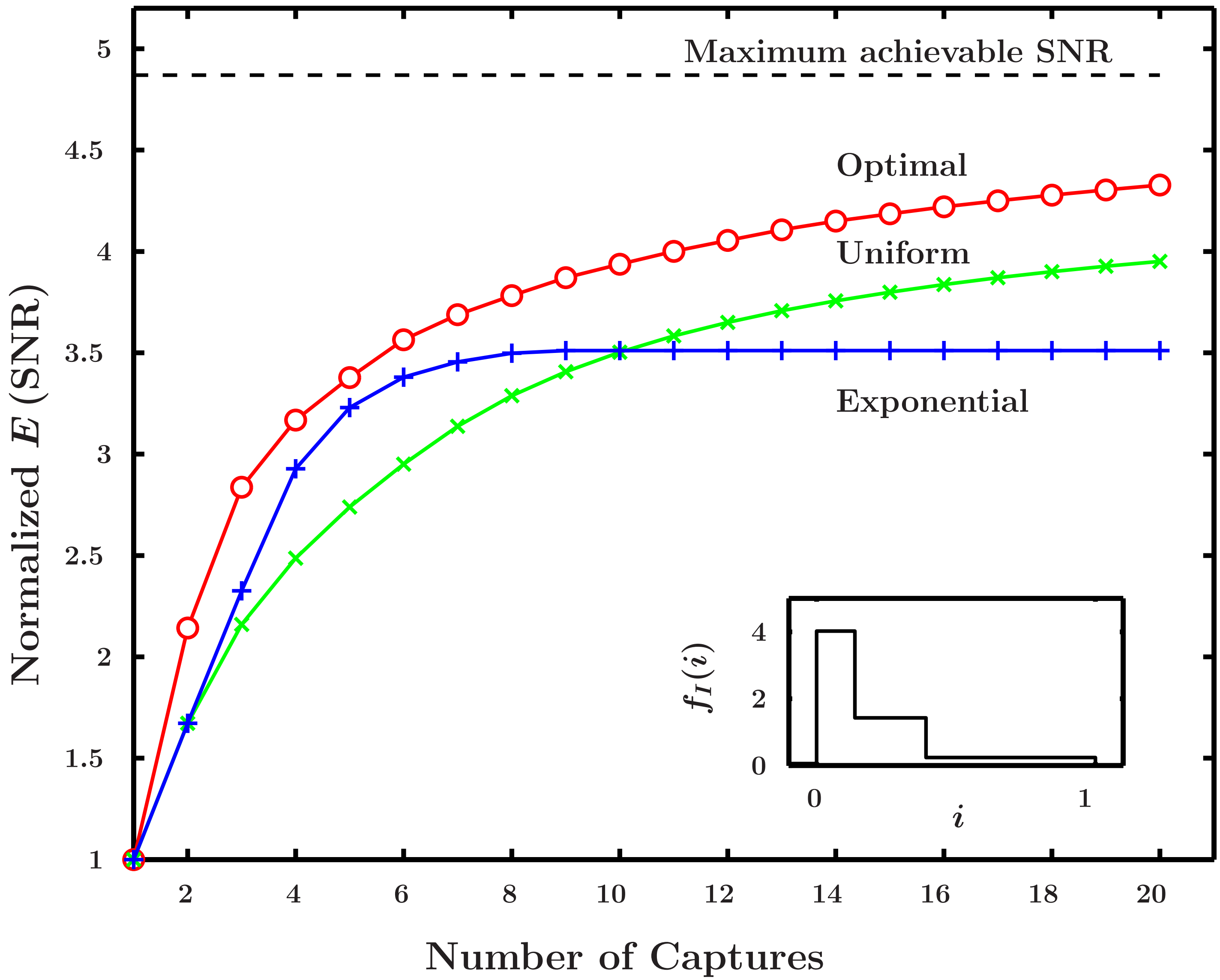

Conclusion

- Presented the first systematic study of optimal selection of capture times in a multiple capture imaging system
- Assumed complete a priori knowledge of scene illumination statistics
- Formulated the scheduling problem as an optimization problem that maximizes $\boldsymbol{E}(\mathrm{SNR})$
- Scheduling problem for uniform pdfs can be solved using convex optimization techniques
- Scheduling problem for piece-wise uniform pdfs can be solved using D.C. optimization techniques
- General pdfs can be approximated with piece-wise uniform pdfs