Photocurrent Estimation from Multiple Non-destructive Samples in a CMOS Image Sensor

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ABSTRACT

CMOS image sensors generally suffer from lower dynamic range than CCDs due to their higher readout noise. Their high speed readout capability and the potential of integrating memory and signal processing with the sensor on the same chip, open up many possibilities for enhancing their dynamic range. Earlier work have demonstrated the use of multiple non-destructive samples to enhance dynamic range, while achieving higher SNR than using other dynamic range enhancement schemes. The high dynamic range image is constructed by appropriately scaling each pixel's last sample before saturation. Conventional CDS is used to reduce offset FPN and reset noise. This simple high dynamic range image construction scheme, however, does not take full advantage of the multiple samples. Readout noise power, which doubles as a result of performing CDS, remains as high as in conventional sensor operation. As a result dynamic range is only extended at the high illumination end. The paper explores the use of linear mean-square-error estimation to more fully exploit the multiple pixel samples to reduce readout noise and thus extend dynamic range at the low illumination end. We present three estimation algorithms: (1) a recursive estimator when reset noise and offset FPN are ignored, (2) a non-recursive algorithm when reset noise and FPN are considered, and (3) a recursive estimation algorithm for case (2), which achieves mean square error close to the non-recursive algorithm without the need to store all the samples. The later recursive algorithm is attractive since it requires the storage of only a few pixel values per pixel, which makes its implementation in a single chip digital imaging system feasible.

1. INTRODUCTION

Dynamic range is a critical figure of merit for image sensors. It is defined as the ratio of the largest non-saturating photocurrent to the smallest detectable photocurrent, typically defined as the standard deviation of the noise under dark conditions. CMOS image sensors generally suffer from lower dynamic range than CCDs due to their higher readout noise, and thus higher noise under dark conditions. However, the high speed non-destructive readout capability of a CMOS image sensor and the ability to integrate memory and signal processing with the sensor on the same chip, open up many possibilities for enhancing its dynamic range. Earlier work [1,2] have demonstrated the use of multiple capture to enhance image senor dynamic range. The idea is to capture several images at different times within the normal exposure time — shorter exposure time images capture the brighter areas of the scene while longer exposure time images capture the darker areas of the scene. The captured images are then combined into a single high dynamic range image by appropriately scaling each pixel's last sample before saturation. Conventional CDS is used to reduce reset and offset FPN. It was shown that this multiple capture scheme achieves higher SNR than other dynamic range enhancement schemes.[3] The scheme does not take full advantage of the multiple pixel samples, however. Readout noise, whose power is doubled as a result of performing CDS, remains as high as for conventional sensor operation. As a result dynamic range is only extended at the high illumination end.

In this paper we explore the use of linear mean-square-error (MSE) estimation[4] to more fully exploit the multiple pixel samples. The motivation is to reduce readout noise and thus extend dynamic range at the low illumination end. Dynamic range is also extended at the high end by considering only pixel samples before saturation. We present three estimation algorithms.

1. A recursive algorithm when reset noise and offset FPN are ignored. In this case only the latest estimate and the new sample are needed to update the photocurrent estimate.
2. A non-recursive algorithm when reset noise and FPN are considered.
3. A recursive estimator for case (2), which is shown to yield mean square error close to the non-recursive algorithm without the need to store all the samples.

The later recursive algorithm is attractive since it requires the storage of only a constant number of values per pixel. Since it is also completely local, operating on each pixel output separately, it is ideally suited for implementation in a single chip digital imaging system.[5]

The rest of the paper is organized as follows. In section 2 we describe the sensor noise model used in the derivation of the photocurrent estimation algorithms. In section 3 we describe the three estimation algorithms. In section 4 we present simulation results that demonstrate the dynamic range and SNR improvements using our algorithms.

2. SENSOR MODEL

Figure 1 depicts a simplified pixel model and the charge $Q(t)$ versus time t under different lighting conditions. During capture, each pixel converts incident light into photocurrent i_{ph}, for $0 \leq t \leq T$, where T is the exposure time. The photocurrent is integrated onto a capacitor and the charge $Q(T)$ (or voltage) is read out at the end of exposure time T. Dark current i_{dc} and additive noise corrupt the photocharge. The noise can be expressed as the sum of three independent components, (i) shot noise $U(T) \sim \mathcal{N}(0, q(i_{ph} + i_{dc})T)$, where q is the electron charge, (ii) readout circuit noise $V(T)$ (including quantization noise) with zero mean and variance σ_V^2, and (iii) reset and FPN noise C with zero mean and variance σ_C^2, which is the same for all multiple captures. Thus the output charge from a pixel can be expressed as

$$Q(T) = \begin{cases} (i_{ph} + i_{dc})T + U(T) + V(T) + C, & \text{for } Q(T) \leq Q_{sat} \\ Q_{sat}, & \text{otherwise} \end{cases}$$

where Q_{sat} is the saturation charge, also referred to as *well capacity*. The SNR can be expressed as

$$\text{SNR}(i_{ph}) = 10 \log_{10} \frac{(i_{ph}T)^2}{q(i_{ph} + i_{dc})T + \sigma_V^2 + \sigma_C^2}$$

Note that SNR increases with i_{ph}, first at 20dB per decade when reset and readout noise dominate, then at 10dB per decade when shot noise dominates. SNR also increases with T. Thus it is always preferred to have the longest possible exposure time. Saturation and motion blur, however, impose practical limits on exposure time.

3. PHOTOCURRENT ESTIMATION FROM MULTIPLE SAMPLES

The multiple capture scheme as originally described[1,2] enhances dynamic range at the high illumination end as illustrated in Figure 1. In the high light case, the pixel saturates before the end of integration time T, and thus its true photocurrent value cannot be faithfully reproduced. Using the multiple capture scheme the pixel charge is non-destructively read out at $t = \{0, \tau, \ldots, T\}$ and the pixel photocurrent is estimated by appropriately scaling the difference between the last sample before saturation and the sample at 0. The subtraction of the sample at 0 reduces reset noise and offset FPN. For example, for the high light case in Figure 1, the sample at 0 is subtracted from the third sample and the difference is scaled by 3τ. The justification for ignoring all other samples before saturation is that the last sample has the highest SNR. This is quite acceptable at the high illumination end, where shot noise dominates, but results in no dynamic range enhancement at low illumination where no saturation occurs and readout noise dominates.

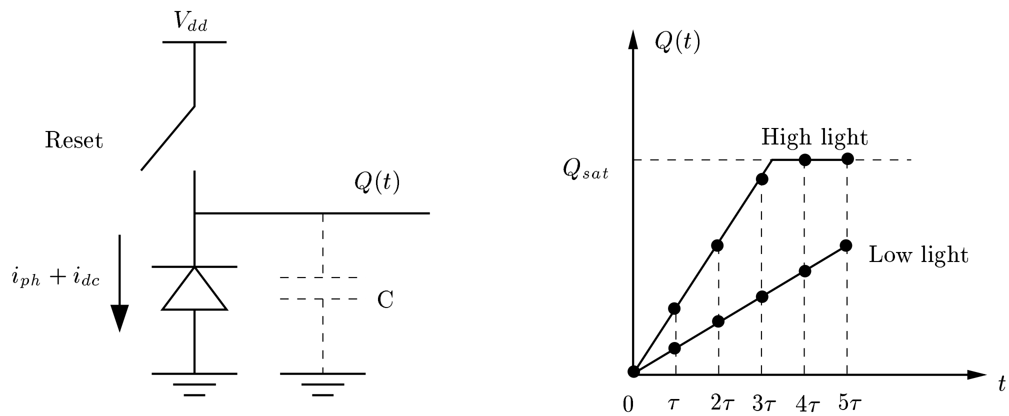

Figure 1. Simplified photodiode pixel model and the photocharge $Q(t)$ vs. time t under different light intensity

Dynamic range at the low illumination end can be enhanced using the multiple samples by appropriately *averaging* the samples before saturation to reduce readout noise. In order for this averaging to be effective, however, the sensor signal and noise model discussed in the previous section must be taken into consideration. In this section we use linear MSE estimation to derive the optimal weights to be used in the averaging. We assume $n+1$ pixel charge samples Q_k captured at times $0, \tau, 2\tau, \ldots, n\tau = T$ and define the pixel current $i = i_{ph} + i_{dc}$. The kth charge sample is thus given by

$$Q_k = ik\tau + \sum_{j=1}^{k} U_j + V_k + C, \text{ for } 0 \le k \le n,$$

where V_k is the readout noise of the kth sample, U_j is the shot noise generated during the time interval $((j-1)\tau, j\tau]$, and the U_js, V_k, C are independent zero mean random variables with

$$\begin{aligned} E(V_k^2) &= \sigma_V^2 > 0, \text{ for } 0 \le k \le n, \\ E(U_j^2) &= \sigma_U^2 = qi\tau, \text{ for } 1 \le j \le k, \text{ and} \\ E(C^2) &= \sigma_C^2. \end{aligned}$$

At time $k\tau$, we wish to find the best unbiased linear MSE estimate, $\hat{I}_k$, of i given $\{Q_0, Q_1, \ldots, Q_k\}$, *i.e.*, we wish to find $b_0, b_1, \ldots, b_k$ such that

$$\hat{I}_k = \sum_{j=0}^{k} b_j Q_j,$$

minimizes

$$\Phi_k^2 = E(\hat{I}_k - i)^2.$$

subject to

$$E(\hat{I}_k) = i.$$

In the following subsections, we present estimation algorithms for three cases, (1) when reset noise and offset FPN are ignored, (2) when reset noise and FPN are considered, and (3) a recursive estimator for case (2), which is shown to yield mean square error close to algorithm in (2) without the need to store all the samples.

3.1. Estimation Ignoring Reset Noise and FPN

Here we ignore reset noise and offset FPN, *i.e.*, set $C = 0$. Even though this case does not correspond to any practical sensor situation, the simplification makes it possible to cast the optimal estimation algorithm in a recursive form. As we shall see in the next subsection, this is not the case in general.

To derive the best estimate, define the pixel current samples as

$$\tilde{I}_k = \frac{Q_k}{k\tau} = i + \frac{\sum_{j=1}^{k} U_j}{k\tau} + \frac{V_k}{k\tau}, \text{ for } 1 \leq k \leq n$$

At time $k\tau$, we wish to find the best unbiased linear mean square estimate of the parameter i given $\{\tilde{I}_1, \tilde{I}_2, \ldots, \tilde{I}_k\}$, *i.e.*, coefficients $a_1, a_2, \ldots, a_k$ such that

$$\hat{I}_k = \frac{1}{g_k} \sum_{j=1}^{k} a_j \tilde{I}_j,$$

where $g_k = \sum_{j=1}^{k} a_j$, minimizes

$$\Phi_k^2 = E(\hat{I}_k - i)^2,$$

subject to

$$E(\hat{I}_k) = i$$

The mean square error (MSE) Φ_k^2 is given by

$$\begin{aligned} \Phi_k^2 &= E(\hat{I}_k - i)^2 \\ &= \frac{1}{g_k^2} \left(\sum_{j=1}^{k} (\sum_{l=j}^{k} \frac{a_l}{l})^2 \frac{\sigma_U^2}{\tau^2} + \sum_{j=1}^{k} (\frac{a_j}{j})^2 \frac{\sigma_V^2}{\tau^2} \right) \end{aligned}$$

The optimal a_js can be found using the optimality conditions

$$\frac{\partial \Phi_k^2}{\partial a_j} = 0, \text{ for } 1 \leq j \leq k,$$

and the unbiased estimator constraint.

The optimal estimate can be expressed in the recursive form

$$\hat{I}_k = \hat{I}_{k-1} + h_k(\tilde{I}_k - \hat{I}_{k-1}),$$

where the gain h_k is given by

$$h_k = \frac{a_k}{g_{k-1} + a_k}$$

and the a_ks are given by

$$a_k = k(1 + \frac{a_{k-1}}{k-1} + \frac{\sigma_U^2}{\sigma_V^2} w_{k-1}),$$

for $w_k = \sum_{j=1}^{k} \frac{a_j}{j}$.

The MSE Φ_k^2 can also be expressed in a recursive form as

$$\Phi_k^2 = \frac{g_{k-1}^2}{g_k^2} \Phi_{k-1}^2 + \frac{1}{g_k^2}((2a_k g_{k-1} + a_k^2) \frac{\sigma_U^2}{k\tau^2} + a_k^2 \frac{\sigma_V^2}{(k\tau)^2})$$

The initial conditions for computing the estimate and its MSE are given by

$$\begin{aligned} a_1 &= 1, \\ \hat{I}_1 &= \tilde{I}_1, \text{ and} \\ \Phi_1^2 &= \frac{\sigma_U^2}{\tau^2} + \frac{\sigma_V^2}{\tau^2} \end{aligned}$$

To compute the estimate $\hat{I}_{k+1}$ and the MSE Φ_{k+1}^2 we need to know $\sigma_U^2 = qi\tau$, which means that we need to know i! We solve this problem by using the latest estimate of i, $\hat{I}_k$, to approximate σ_U^2. We found that this approximation yields MSE very close to the optimal case, *i.e.*, when i is known.

3.2. Estimation Considering Reset noise and FPN

With reset noise and offset FPN taken into consideration, we redefine $\tilde{I}_k$ as

$$\tilde{I}_k = \frac{Q_k - wQ_0}{k\tau}, \text{ for } 1 \le k \le n.$$

The weight w is obtained by solving for the optimal b_0 and is given by

$$w = \frac{\sigma_C^2}{\sigma_C^2 + \sigma_V^2}.$$

Note that $\tilde{I}_k$ corresponds to an estimate with a *weighted* CDS operation. The weighting has the effect of reducing the additional readout noise due to CDS.

The pixel current estimate given the first k samples can be expressed as

$$\hat{I}_k = \mathbf{A}_k \tilde{\mathbf{I}}_k,$$

where

$$\begin{aligned} \mathbf{A}_k &= [a_1^{(k)} \; a_2^{(k)} \; \dots \; a_2^{(k)}], \text{ and} \\ \tilde{\mathbf{I}}_k &= [\tilde{I}_1 \; \tilde{I}_2 \; \dots \; \tilde{I}_k]^T. \end{aligned}$$

The optimal coefficient vector $\mathbf{A}_k$ is given by

$$\mathbf{A}_k = -(\frac{\sigma_U^2}{\tau^2} M_k + \frac{\sigma_V^2}{\tau^2}(T_k + wD_kF_k^T))^{-1}\frac{\lambda}{2}L_k,$$

where

$$M_k = \begin{bmatrix} 1 & \frac{1}{2} & \dots & \frac{1}{k} \\ 1 & 1 & \dots & \frac{2}{k} \\ \dots & & & \\ 1 & 1 & \dots & 1 \end{bmatrix}, \; D_k = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \; F_k = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \vdots \\ \frac{1}{k} \end{bmatrix}, \; L_k = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ k \end{bmatrix}, \; T_k = \mathbf{diag}\{F_k\}$$

and λ is the Lagrange multiplier for the unbiased constraint

$$\sum_{j=1}^{k} a_j^{(k)} = 1.$$

It can be shown that the above solution cannot be expressed in a recursive form and thus finding $\hat{I}_k$ requires the storage of the vector $\tilde{\mathbf{I}}_k$ and inverting a $k \times k$ matrix.

3.3. Recursive Algorithm

Now, we restrict ourselves to recursive estimates, *i.e.*, estimates of the form

$$\hat{I}_k = \hat{I}_{k-1} + h_k(\tilde{I}_k - \hat{I}_{k-1}),$$

where again

$$\tilde{I}_k = \frac{Q_k - wQ_0}{k\tau}.$$

The coefficient h_k can be found by solving the equations

$$\begin{aligned} \frac{d\,\Phi_k^2}{d\,h_k} &= \frac{d\,E(\hat{I}_k - i)^2}{d\,h_k} = 0, \text{ and} \\ E\hat{I}_k &= i. \end{aligned}$$

Define the MSE of $\tilde{I}_k$ as

$$\Delta_k^2 = E(\tilde{I}_k - i)^2 = \frac{1}{k^2\tau^2}(k\sigma_U^2 + (1+w)\sigma_V^2)$$

and the covariance between $\tilde{I}_k$ and $\hat{I}_k$ as

$$\begin{aligned}\Theta_k &= E(\tilde{I}_k - i)(\hat{I}_k - i) \\ &= (1-h_k)\frac{k-1}{k}\Theta_{k-1} - \frac{(1-h_k)h_{k-1}}{k(k-1)\tau^2}\sigma_V^2 + h_k\Delta_k^2.\end{aligned}$$

The MSE of $\hat{I}_k$ can be expressed in terms of Δ_k^2 and Θ_k as

$$\Phi_k^2 = (1-h_k)^2\Phi_{k-1}^2 + \frac{2(k-1)(1-h_k)h_k}{k}\Theta_{k-1} - \frac{2h_{k-1}(1-h_k)h_k}{k(k-1)\tau^2}\sigma_V^2 + {h_k}^2\Delta_k^2.$$

To minimize the MSE, we require that

$$\frac{d\,\Phi_k^2}{d\,h_k} = 0,$$

Which gives

$$h_k = \frac{\Phi_{k-1}^2 - \frac{(k-1)}{k}\Theta_{k-1} + \frac{h_{k-1}\sigma_V^2}{k(k-1)\tau^2}}{\Phi_{k-1}^2 - \frac{2(k-1)}{k}\Theta_{k-1} + \frac{2h_{k-1}\sigma_V^2}{k(k-1)\tau^2} + \Delta_k^2}$$

Note that h_k, Θ_k and Φ_k can all be recursively updated.

To summarize, the suboptimal recursive algorithm is as follows.

Set initial parameter and estimate values as follows:

$$\begin{aligned}h_1 &= 1 \\ \tilde{I}_1 &= \frac{(Q_1 - wQ_0)}{\tau} \\ \hat{I}_1 &= \tilde{I}_1 \\ \Delta_1^2 &= \frac{\sigma_U^2 + (1+w)\sigma_V^2}{\tau^2} \\ \Phi_1^2 &= \Delta_1^2 \\ \Theta_1 &= \Delta_1^2\end{aligned}$$

At each iteration, the parameter and estimate values are updated as follows:

$$\begin{aligned}\tilde{I}_k &= \frac{(Q_k - wQ_0)}{k\tau} \\ \Delta_k^2 &= \frac{1}{k^2\tau^2}(k\sigma_U^2 + (1+w)\sigma_V^2) \\ h_k &= \frac{\Phi_{k-1}^2 - \frac{(k-1)}{k}\Theta_{k-1} + \frac{h_{k-1}\sigma_V^2}{k(k-1)\tau^2}}{\Phi_{k-1}^2 - \frac{2(k-1)}{k}\Theta_{k-1} + \frac{2h_{k-1}\sigma_V^2}{k(k-1)\tau^2} + \Delta_k^2} \\ \Theta_k &= (1-h_k)\frac{k-1}{k}\Theta_{k-1} - \frac{(1-h_k)h_{k-1}}{k(k-1)\tau^2}\sigma_V^2 + h_k\Delta_k^2 \\ \Phi_k^2 &= (1-h_k)^2\Phi_{k-1}^2 + 2h_k\Theta_k - {h_k}^2\Delta_k^2. \\ \hat{I}_k &= \hat{I}_{k-1} + h_k(\tilde{I}_k - \hat{I}_{k-1})\end{aligned}$$

Note that to find the new estimate $\hat{I}_k$, only three parameters, h_k, Φ_k and Θ_k, the old estimate $\hat{I}_{k-1}$ and the new sample value $\tilde{I}_k$ are needed. Thus only a small fixed amount of memory per pixel is required.

4. SIMULATION RESULTS

In this section we present simulation results that demonstrate the SNR and dynamic range improvements using the non-recursive algorithm described in Subsection 3.2 compared to those achieved by the recursive algorithm in Subsection 3.3 and the earlier multiple capture scheme.[1,2]

The simulation results are summarized in Figures 2, 3, 4. The sensor parameters assumed in the simulations are as follows.

$$\begin{aligned} Q_{sat} &= 18750 \text{ e-} \\ i_{dc} &= 0.1 \text{ fA} \\ \sigma_V &= 60 \text{ e-} \\ \sigma_C &= 62 \text{ e-} \\ T &= 32 \text{ ms} \\ \tau &= 1 \text{ ms} \end{aligned}$$

Figures 2 and 3 compare the equivalent readout noise RMS and SNR values at low illumination level corresponding to $i_{ph} = 2$ fA as a function of the number of samples k for conventional sensor operation and using the non-recursive and the recursive estimation algorithms. As can be seen in Figure 2, the equivalent readout noise after the last sample is reduced from 86 e- when no estimation is used to 35.8 e- when the non-recursive estimator is used and to 56.6 e- when the recursive estimator is used. Equivalently, as can be seen in Figure 3, SNR increases by 6.6 dB using the non-recursive estimator versus 3.34 dB using the recursive estimator. Also note the drop in the equivalent readout noise RMS due to the weighted CDS used in our algorithms.

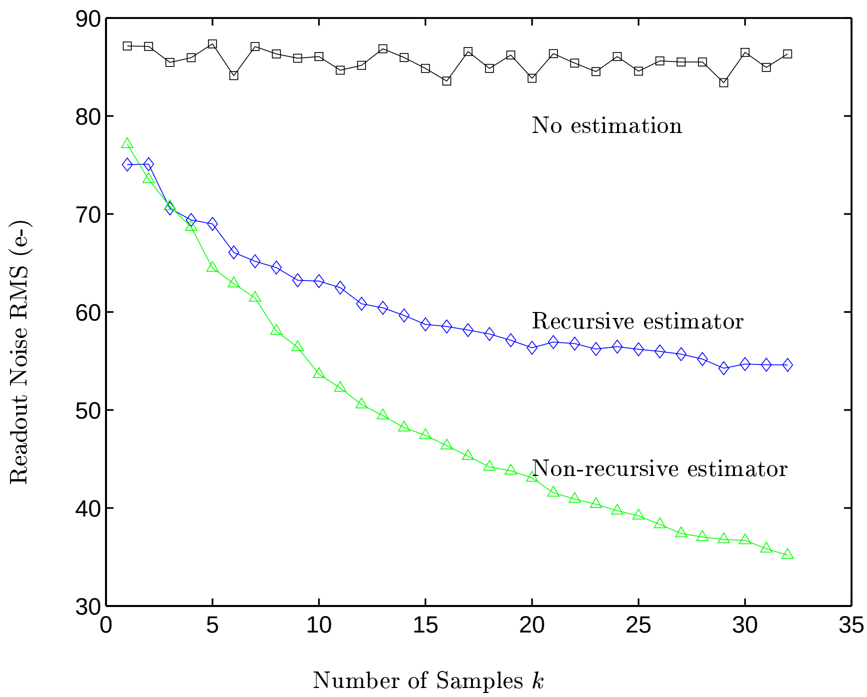

Figure 2. Equivalent Readout noise rms value vs. k.

Figure 4 plots SNR versus i_{ph} using the standard multiple capture scheme[1,2] and using our estimators. Note that using our estimators consistently improves SNR, with the most pronounced improvement being at the low illumination end. More significantly the sensor dynamic range defined as the ratio of the largest

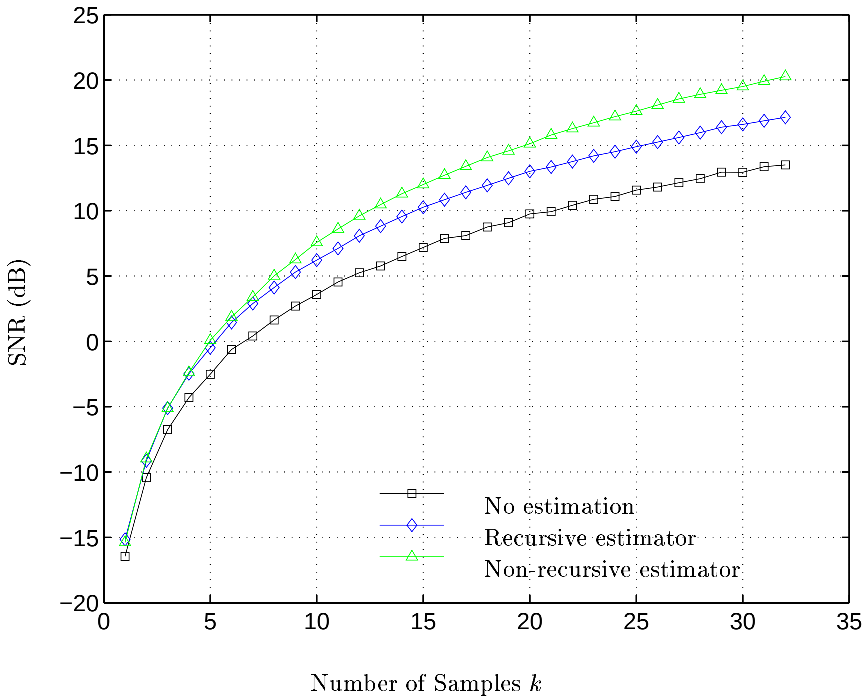

Figure 3. SNR vs. k.

non-saturating photocurrent i_{max} to the smallest detectable photocurrent i_{min} is increased from 73.5 dB using the standard multiple capture scheme to 81.5 dB using the non-recursive estimator and to 77.5 dB using the recursive estimator.

5. CONCLUSION

We presented estimation algorithms that exploit the high speed imaging capability of CMOS image sensors to enhance its dynamic range and SNR beyond the standard multiple capture scheme.[1,2] While the standard multiple capture scheme extends dynamic range only at the high illumination end, our algorithms also extend it at the low illumination end by averaging out readout noise. The non-recursive estimation algorithm presented significantly increases dynamic range and SNR but requires the storage of all frames and performing costly matrix inversions. To reduce the storage and computational complexity we also derived a recursive algorithm. We showed that the dynamic range and SNR improvements achieved using the recursive estimator although not as impressive as using the non-recursive estimator, are quite significant. The recursive algorithm, however, has the important advantage of requiring the storage of only a few pixel values per pixel and modest computational power, which makes its implementation in a single chip digital imaging system quite feasible.

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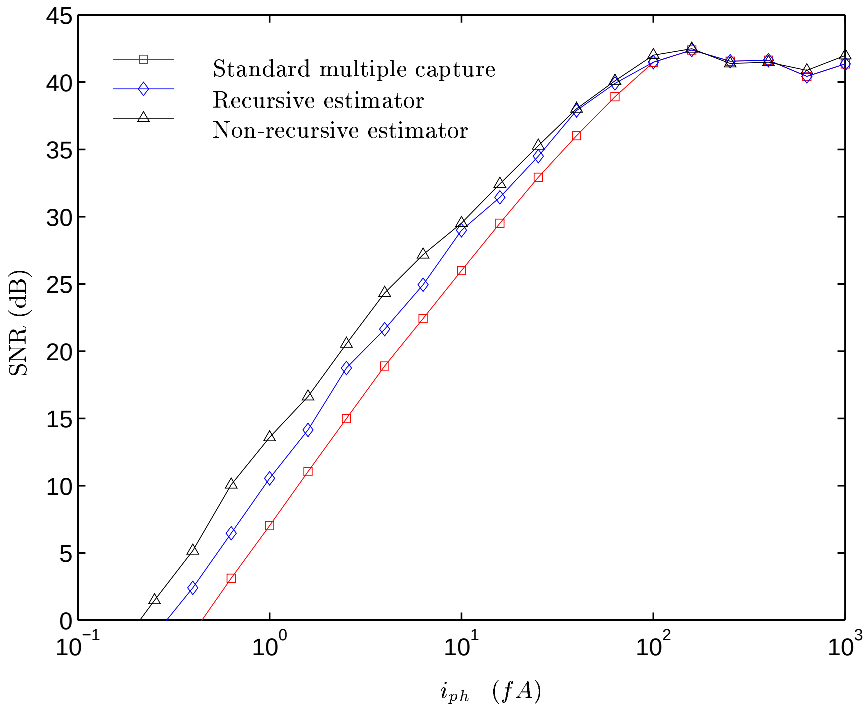

Figure 4. SNR vs. i_{ph}.

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