ADAPTIVE SIGNAL PROCESSING FOR ADAPTIVE CONTROL

by

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Abstract. A few of the well established methods of adaptive signal processing theory are modified and extended in order to address some of the basic issues of adaptive control.

An unknown plant will track an input command signal if the plant is preceded by a controller whose transfer function approximates the inverse of the plant transfer function. An adaptive inverse modeling process can be used to obtain a stable controller, whether the plant is minimum or non-minimum phase. A model-reference version of this idea allows system dynamics to closely approximate desired reference model dynamics. No direct feedback is involved. However the system output is monitored and utilized in order to adjust the parameters of the controller.

The proposed method performs very well in computer simulations of a wide range of stable plants, and it seems to be a promising alternative approach to the design of adaptive control systems.

Keywords. Adaptive control, self-adjusting system, controllers, modeling, transfer functions.

INTRODUCTION

There is a great need for learning-control systems which can adapt to the requirements of plants whose characteristics may be unknown and/or changeable in unknown ways. Two principal factors have hampered the development of adaptive controls:

a) the difficulty of dealing with learning processes embedded in feedback loops.

b) the difficulty in controlling nonminimum-phase plants.

Considerable progress has been made (see for instance works by Powell, 1969; Tse and Athans, 1972; Nakamura and Yoshida, 1973; Astrom and Wittenmark, 1973, 1974, 1980; Landau, 1974, 1976, Martin-Sanchez, 1976). However, interaction between the feedback of the learning process and that of the signal flow path still greatly complicates the analysis which is requisite to the design of dependable control systems.

In this paper we continue with the development of an alternative approach, which was first presented by B. Widrow and his students 1978, 1981, and B. D. O. Anderson, 1981, which circumvents many of the difficulties that have been encountered with the previous forms of adaptive control. The basic idea is to create a good transversal filter model of the plant, then to utilize it in order to obtain an inverse (or delayed inverse) of the plant. This inverse can be used as an open loop controller of the system. Since such a controller is realized as a transversal filter, the stability of the system is assured. Moreover it can be shown that, if one is willing to allow a delay in the response of the control system, excellent control of the plant dynamics can be achieved, even for nonminimum phase plants.

In this paper, the basic principles of the proposed approach will be discussed and computer simulations will be presented in order to illustrate their potential. It should be mentioned that additional intensive research has been conducted in order to address the issues of

a) cancelling the plant noise by feeding appropriate signal at the plant input

b) facilitating the modeling process by employing dither signals when sufficient ambient signal activity is lacking

c) adaptive control of MIMO (multiple input, multiple output) systems

d) detailed quantitative performance and stability analysis.

Progress in all these areas indicates and verifies the validity of the general approach, although the details of this research will not be included here as they go beyond the scope of the current presentation.

ADAPTIVE FILTERING

A schematic representation of an adaptive filter is depicted in Fig. 1. The filter has an input x_j, an output y_j, and it requires a special training signal called the "desired response" d_j. The error ϵ_j is the difference between the desired and actual output responses. The filter is assumed to be transversal and its weights $w_{1j}, ..., w_{nj}$ are adapted in order to minimize the expected square of the error ϵ_j. Various adaptation algorithms can be utilized for that purpose. Here we will employ the LMS steepest descent algorithm of Widrow and Hoff, 1960, which is well known in the literature (see for instance later works by Widrow and others 1975, 1976).

PLANT MODELING

To illustrate an application of the LMS adaptive filter and to show by example how one obtains an input and a desired response in a control environment, consider the direct modeling of an unknown plant as shown in Fig. 2. When given the same input signal as that of an unknown plant, the adaptive model self-adjusts to cause its output to be a best least squares fit to the actual plant output. The unknown plant may have both poles and zeros, but the adaptive transversal filter can only realize "zeros." (The word zeros is in quotes because the adaptive filter is time variable and does not strictly have a transfer function. In

a quasi-static sense, the adaptive filter can be thought to have "instantaneous zeros" corresponding to the zeros that would exist if the weights were frozen at their instantaneous values.) However with a sufficient number of weights, an adaptive transversal filter can achieve a close fit to an unknown plant having many poles and zeros.

PLANT INVERSE MODELING

The inverse model of the unknown plant could be formed as shown in Fig. 3. The adaptive filter input is the plant output. The filter is adapted to cause its output to be a best least squares fit to the plant input. A close fit implies that the cascade of the unknown plant and the LMS filter have a "transfer function" of essentially unit value. Close fits have been achieved by adaptive transversal inverse filters even when the unknown plant had many poles and zeros.

INVERSE MODELING OF NONMINIMUM PHASE PLANTS

If the plant itself is stable, all of its poles lie in the left half of the s-plane. But some of its zeros could lie in the right half plane, and then the plant would be nonminimum phase. The inverse of the minimum phase plant would have all of its poles in the left half plane, and there would be no problem with stability of the inverse. The nonminimum phase plant would have zeros in the right half plane and stability of the inverse would be an important issue. However, it can be shown that stable inverses for nonminimum phase plants could always be constructed if one were permitted noncausal two-sided impulse responses. Furthermore, with suitable time delays, causal approximations to delayed versions of noncausal impulse responses are realizable. Thus, by allowing a delay in the modeling process (as illustrated in Fig. 3), one can obtain approximate delayed inverse models to minimum phase and nonminimum phase plants. It is not necessary to know a priori whether the plant is or is not minimum phase. However, some knowledge of plant characteristics would be helpful when choosing the delay Δ and the length of the transversal filter used for inverse modeling.

ADAPTIVE INVERSE CONTROL SCHEME

Using a stable delayed inverse, control is accomplished as illustrated in Fig. 4. The controller is a copy of the inverse model. The command input i_j, the desired output for the plant, is applied as an input to the controller.

The controller output is the driving function for the plant. If the controller were an exact delayed plant inverse, the plant output, assuming no noise, would be an exact copy of the input reference command, but delayed, i.e.,

$$y_j = i_{j-\Delta}$$

A step change in the command input would cause a step change in the plant output after a delay of Δ seconds. In order to illustrate this idea, computer simulations were performed. A nonminimum phase plant was controlled. Its impulse response is depicted in Fig. 5a. This stable underdamped plant has a small transport delay. In order to find the inverse, the scheme of Fig. 3 was used to adapt a transversal filter having 40 weights. Since the plant is nonminimum phase, a good (low error) causal inverse cannot be obtained. Hence for $\Delta=0$, the error power was close to the input power. However when the delay Δ was increased, the error power decreased indicating that very good plant inverses were obtained. Figure 5b shows the error power as a function of the modeling delay Δ. For $\Delta=26$, the error power decreased to below 5% of the input signal power. For this value of Δ, the best plant inverse had the impulse response shown in Fig. 5c. Connecting this as a controller in cascade with the plant, in the manner presented in Fig. 4, the overall impulse response was as shown in Fig. 5d. Clearly the behavior of the entire system closely approximated that of a pure delay. In Fig. 6b the step response of the control system is presented, and it may be compared to the ideal step response of Fig. 6a.

MODEL REFERENCE ADAPTIVE CONTROL SYSTEM

Sometimes it is desired that the plant output track not the command input itself but a delayed or smoothed version of the command input. The system designer would generally know the smoothing characteristic to be used. A smoothing model can be readily incorporated into the adaptive inverse control concept, as illustrated in Fig. 7. The smoothing model is usually designated as the "reference model." [See for instance Landau, 1974.] Thus the system of Fig. 7 is a general model-reference adaptive inverse control system. The system of Fig. 4 can be viewed as a special case, when the reference model equals the delay Δ.

OFF-LINE MODEL REFERENCE INVERSE CONTROL

If the plant inverse model has enough weights and if the reference model contains enough delay, the

approach of Fig. 7 allows excellent control of plant dynamics (as demonstrated by the above simulation). However plant noise causes a severe degradation in the performance of this system. Indeed, any noise present in the output of the plant will automatically enter the adaptive inverse modeling process. Therefore, as was pointed by Widrow and others, 1981; the transfer function of the plant inverse, a Wiener solution, will be biased and this bias in turn will cause an erroneous control and deterioration in system performance. To resolve this difficulty, the system of Fig. 8 can be used.

The control system of Fig. 8 works in the following manner. A model $\hat{P}(z)$ of the plant $P(z)$ is formed using the methods mentioned above. $\hat{P}(z)$ is a Wiener solution that is not biased by plant noise. An "off-line" process can then be used to obtain a controller $\hat{C}(z)$ from $\hat{P}(z)$ and the reference model $M(z)$. This process, which could be an adaptive one, adjusts $\hat{C}(z)$ to cause the output of the cascade of $\hat{P}(z)$ and $\hat{C}(z)$ to be a best least squares match to the output of the reference model $M(z)$ when both the cascade and the reference model are driven simultaneously by a synthetic "modeling signal" having an appropriate spectral character. The process for finding $\hat{C}(z)$ could also be non-adaptive, since $\hat{C}(z)$ is deterministically related to $\hat{P}(z)$ and $M(z)$ for any specified modeling signal spectrum. Now given $\hat{C}(z)$, an exact digital copy can then be used as a controller, as shown in Fig. 8. The result is a controller and plant having a cascaded dynamic response which closely approximates the dynamic response of the reference model.

The off-line process of Fig. 8 forms a model-reference inverse of the plant model $\hat{P}(z)$. We have used the model $\hat{P}(z)$ rather than the plant $P(z)$ because the output of the real $P(z)$ is generally corrupted by plant noise. However, since $\hat{P}(z)$ does not perfectly match $P(z)$ at all times, use of $\hat{P}(z)$ in determination of $\hat{C}(z)$ causes errors in $\hat{C}(z)$. However these errors can be limited by slowing the adaptation rate and thus decreasing the error in the plant estimation $\hat{P}(z)$.

CONCLUSION

A method for adaptive inverse control unbiased by additive plant noise has been introduced. The technique is easy to implement and exhibits robust, predictable behavior. Intensive research has been conducted in this area in order to enhance the potential capabilities of the proposed approach and to perform detailed analyses of the expected behavior. The results of this additional research are now being prepared for publication.

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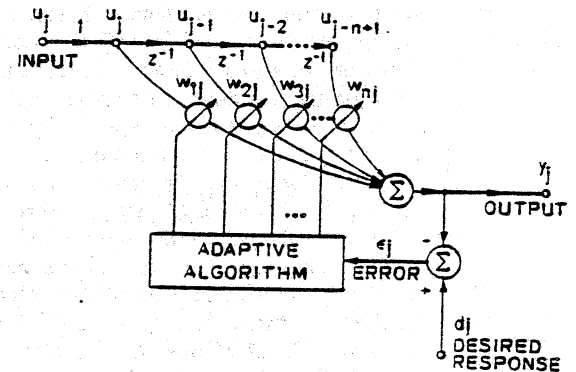
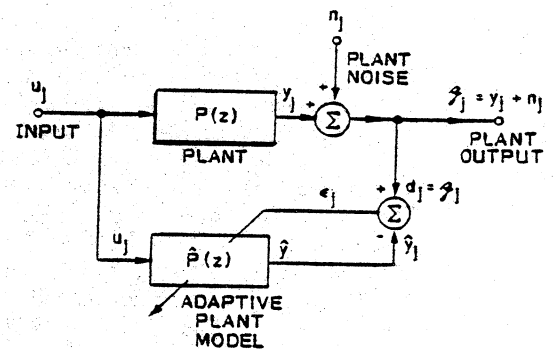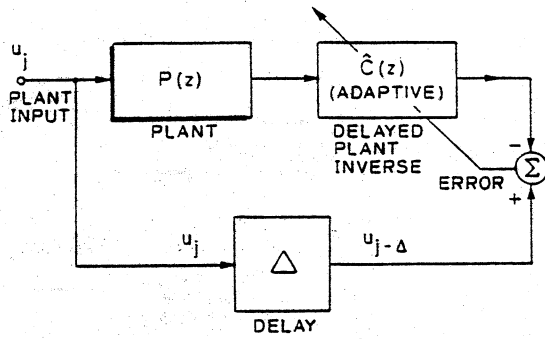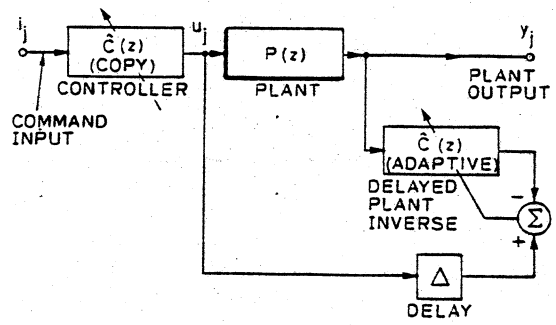FIG. 1. AN ADAPTIVE FILTER.

FIG. 2. ADAPTIVE MODELING.

FIG. 3.   DELAYED INVERSE MODELING.
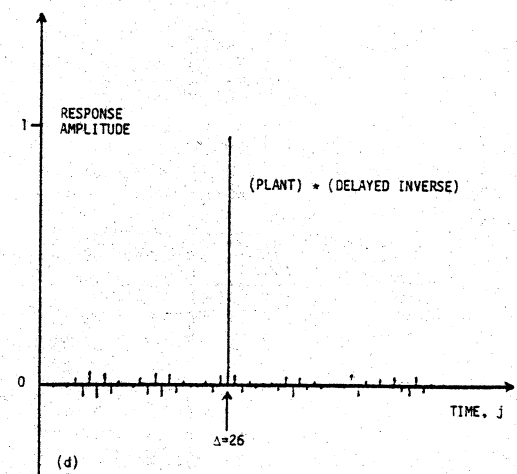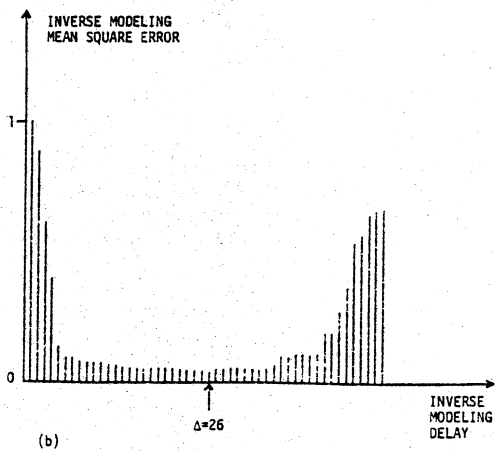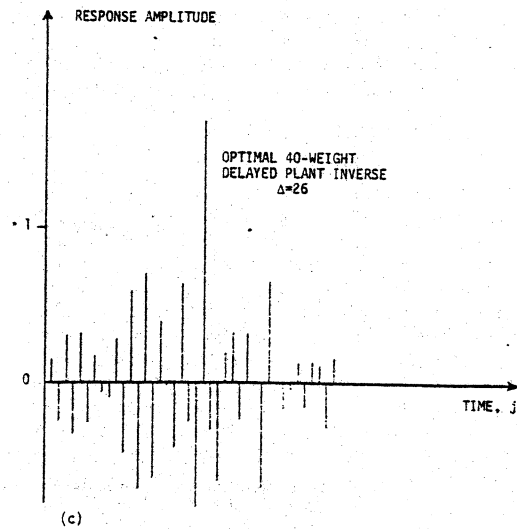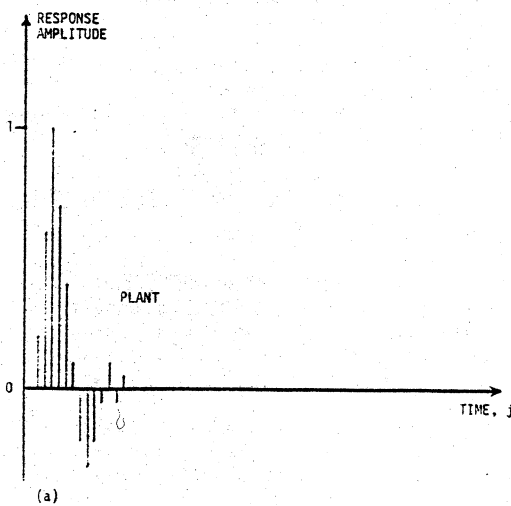

FIG. 4.   AN ADAPTIVE INVERSE
CONTROL SYSTEM.

FIG. 5.   IMPULSE RESPONSE OF PLANT AND
OPTIMIZED 40-WEIGHT CONTROLLER.

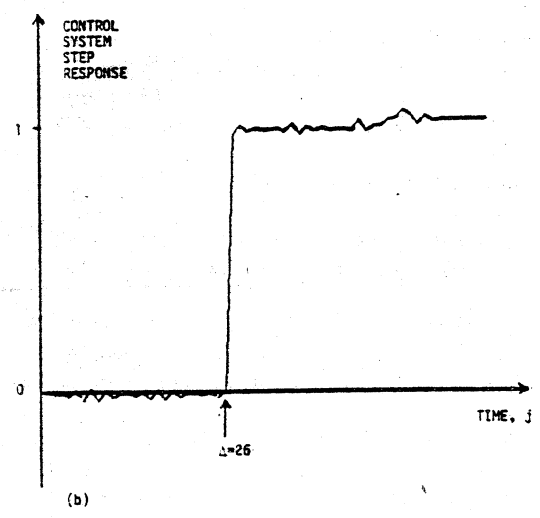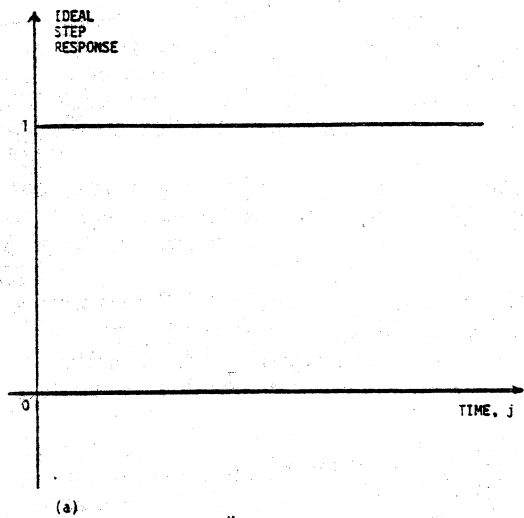IDEAL
STEP
RESPONSE

1

0   TIME, j

(a)

CONTROL
SYSTEM
STEP
RESPONSE

1

0   TIME, j

Δ=26

(b)

FIG. 6.   STEP RESPONSE OF CONTROL SYSTEM
          (PLANT CASCADED WITH 40-WEIGHT
          INVERSE CONTROLLER).

i_j → Ĉ(z) (COPY) CONTROLLER → u_j → P(z) PLANT → $\hat{y}_j$ PLANT OUTPUT

n_j PLANT NOISE

Σ (+ +)

ϵ_j ERROR → Σ (+ −)

P̂(z) (ADAPTIVE) PLANT MODEL

COMMAND INPUT

MODELING NOISE

P̂(z) (COPY) → Ĉ(z) (ADAPTIVE) → Σ (− +) ERROR

M(z) REFERENCE MODEL

OFF-LINE ADAPTIVE PROCESS
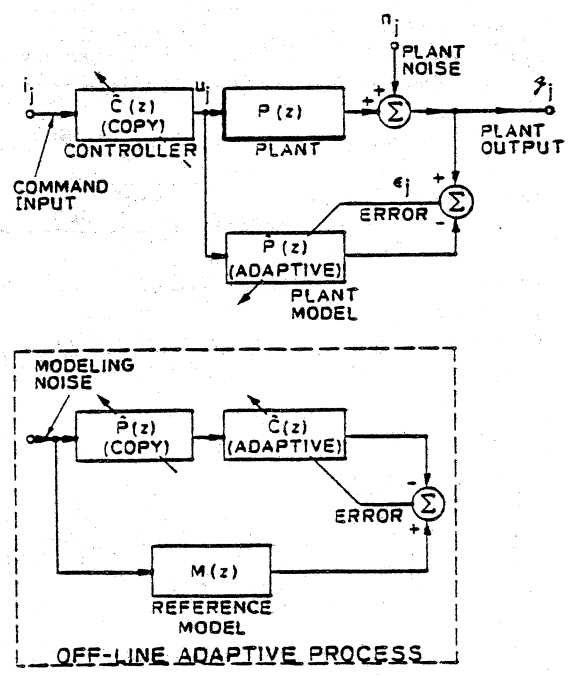
FIG. 8.   A MODEL REFERENCE ADAPTIVE
          CONTROL SYSTEM FOR A NOISY PLANT.

i_j → Ĉ(z) (COPY) CONTROLLER → u_j → P(z) PLANT → y_j PLANT OUTPUT

COMMAND INPUT
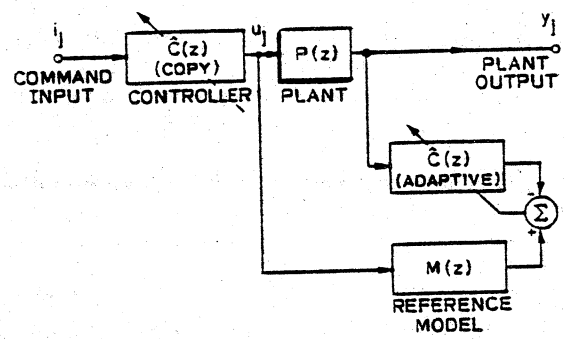
Ĉ(z) (ADAPTIVE)

Σ

M(z) REFERENCE MODEL

FIG. 7.   A MODEL REFERENCE ADAPTIVE
          INVERSE CONTROL SYSTEM.