3.15 THE CAPACITY OF THE RELAY CHANNEL

Consider the following seemingly simple discrete memoryless relay channel:

\[ X \xrightarrow{C_0} Y_1 \xrightarrow{} Y_2 \]

Here \( Y_1, Y_2 \) are conditionally independent and conditionally identically distributed given \( X \), that is, \( p(y_1, y_2 \mid x) = p(y_1 \mid x) \cdot p(y_2 \mid x) \). Also, the channel from \( Y_1 \) to \( Y_2 \) does not interfere with \( Y_2 \). A \((2^nR, n)\) code for this channel is a map \( x : 2^nR \rightarrow X^n \), a relay function \( r : Y_1^n \rightarrow 2^{nC_0} \), and a decoding function \( g : 2^{nC_0} \times Y_2^n \rightarrow 2^nR \). The probability of error is given by

\[ P^{(n)}_e = P\{ g(r(y_1), y_2) \neq W \}, \]

where \( W \) is uniformly distributed over \( 2^nR \) and

\[ p(w, y_1, y_2) = 2^{-nR} \prod_{i=1}^{n} p(y_{1i} \mid x_i(w)) \prod_{i=1}^{n} p(y_{2i} \mid x_i(w)). \]

Let \( C(C_0) \) be the supremum of the achievable rates \( R \) for a given \( C_0 \), that is, the supremum of the rates \( R \) for which \( P^{(n)}_e \) can be made to tend to zero.

We note the following facts:
1. \[ C(0) = \sup_{p(x)} I(X; Y_2). \]
2. \[ C(\infty) = \sup_{p(x)} I(X; Y_1, Y_2). \]
3. \( C(C_0) \) is a nondecreasing function of \( C_0 \).

What is the critical value of \( C_0 \) such that \( C(C_0) \) first equals \( C(\infty) \)?

REFERENCES